

Binomial Random Variable

$$P(x) = \binom{n}{x} p^x q^{n-x}, \quad x = 0, 1, 2, \dots, n$$

X : is a binomial random variable that represent the no. of Success's in n -trials.

n: No. of Trials.

P : Probability of success in a single trial.
 Q : $(1-P)$ probability of fail.

Ex: In an experiment 10% of people passes. Suppose that 4 persons are selected at random.

- a) Find the probability that none of the four persons passes the test.
- b) _____ three persons passes.
- d) Derive Formula for $P(x)$, the probability distribution fn. of the binomial random variable x

4) Non-passes

$$P(\text{FFFF}) = (0.9)^4$$

$$X=0 \quad n=4 \quad p=0.1 \quad q=0.9$$

$$P(0) = C_0^4 P^0 q^4 = 1 * 1 * (0.9)^4$$

b) 3-passes $\rightarrow X=3$

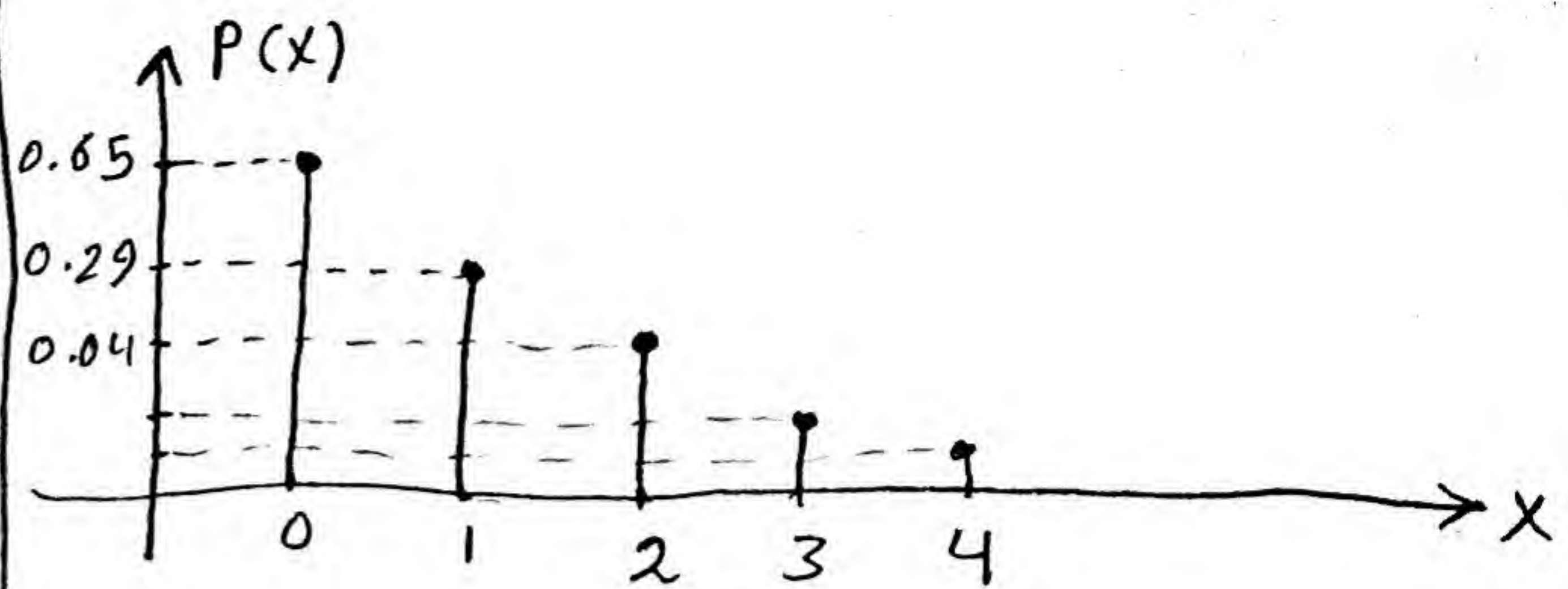
$$P(3) = C_3^4 p^3 q^1$$
$$= 4 * (0.1)^3 * (0.9)$$

d)

x	0	1	2	3	4
$P(x)$	0.65	0.29	0.048	0.0036	0.0001

$$p(x) = C_x p^x q^{n-x}$$

$$n=4, p=0.1, q=0.9$$



For Binomial Random Variable

$$\mu = \eta p$$

$$\text{Var} = npq = \sigma^2$$

$$S.D = \sqrt{\text{Var}} = \sqrt{npq}$$

Sheet 5

Q3] Prove that For any random Variable

i) $E(aX+b) = aE(X) + b$

$$E(ax+b) = \int_{-\infty}^{\infty} (ax+b) p(x) dx$$

$$= a \int_{-\infty}^{\infty} x p(x) dx + b \int_{-\infty}^{\infty} p(x) dx$$

$$\downarrow \qquad \qquad \qquad \downarrow$$

$$E(x) \qquad \qquad \qquad 1$$

$$= a E(x) + b$$

7 Given a formula for $P(x)$ for a binomial random variable with $n = 7$ and $p = 0.2$

$$P(x) = \binom{7}{x} (0.2)^x (0.8)^{7-x}$$

8 Consider the following

$$P(x) = \binom{5}{x} (0.7)^x (0.3)^{5-x}$$

d) $n = 5$ $p = 0.7$

Graph $P(x)$

$$P(0) = \frac{1}{2}$$

$$P(1) = -$$

$$P(\emptyset) = -$$

X	0	1	---	5
P(x)				

$$d) \mu = np = 5 \cdot 0.7$$

$$S.D = \sqrt{npq} = \sqrt{5 * 0.7 * 0.3}$$

Report \rightarrow 10

The probability of exactly k success in n repeated trials

$$b(k; n, p) = \binom{n}{k} p^k q^{n-k}$$

(11) A fair coin \rightarrow 6 times
head \rightarrow Success

Find

- i) Probability that exactly 2 heads
- ii) // " at least 4 heads
- iii) // " no heads
- iv) // " at least one head

$$n = 6$$

$$p = 0.5$$

$$q = 0.5$$

$$i) P(x) = \binom{n}{x} p^x q^{n-x}$$

$$P(2) = \binom{6}{2} (0.5)^2 (0.5)^4 = \checkmark$$

$$ii) P(4) + P(5) + P(6)$$

$$= \binom{6}{4} (0.5)^4 (0.5)^2 + \binom{6}{5} (0.5)^5 (0.5)^1 + \binom{6}{6} (0.5)^6 (0.5)^0 = \checkmark$$

$$ii) P(0) = \binom{6}{0} = \checkmark$$

$$iv) P(1) + P(2) + \dots + P(6)$$

$$= 1 - b(0; 6; 0.5)$$

$$= 1 - P(0) = \checkmark$$